Thornton & Marion 5th Edition Problem 2 – 43

A particle is under the influence of a force F = -kx + $\frac{k}{\alpha^2}x^3$ where k and α are constants and k is always positive. Determine the potential energy and discuss the motion. What happens when E = $\frac{1}{4}k\alpha^2$.

Define the function:

```
Force[x_] = -k*x + k/α^2*x^3;
```

Integrate to find the potential energy:

```
PotEnergy = -∫ Force[x] dx
```

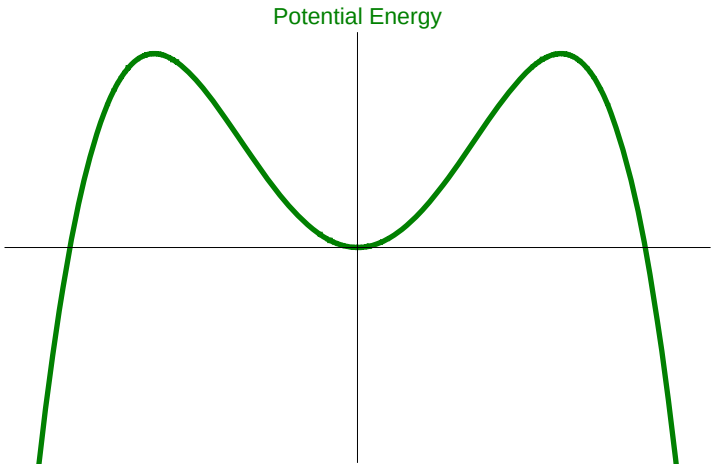
$$\frac{k\,x^2}{2} - \frac{k\,x^4}{4\,\alpha^2}$$

Define the constants with random values:

```
k = 2;
α = 3;
```

Plot U to show maxima and minima:

```
pPotEnergy =
  Plot[PotEnergy, {x, -5, 5}, BaseStyle → {FontFamily → Helvetica, FontSize → 12, FontColor → RGBColor[0, 0.5, 0]},
    Ticks → None, PlotRange → {-5, 5}, PlotPoints → 100,
    PlotStyle → {{RGBColor[0, 0.5, 0], Thickness[0.0075]}}, PlotLabel → "Potential Energy"]
```



Find the maxima and minima of the potential energy. Since these occur where $\frac{dU}{dx}$= -F = 0, find the values for this.

```
Clear[k, α]
Solve[-k*x + k/α^2*x^3 == 0, x]
```

{{x → 0}, {x → -α}, {x → α}}

The potential energy, $U(x) = \frac{k x^2}{2} - \frac{k x^4}{4 \alpha^2}$, at the maxima, $U(x_{max}) = U(\pm\alpha)$ and $U(0)$ are

$U(\pm\alpha) = \frac{k \alpha^2}{2} - \frac{k \alpha^4}{4 \alpha^2} = \frac{k \alpha^2}{4}$ and

$U(0) = 0$.

Thus when $E = T + U = \frac{k \alpha^2}{4}$, $E = U(\pm\alpha)$ so that $T = O$ at these points. Thus the particle is bound as shown in the plot below for the values of k and α above.

```
k = 2;
α = 3;
k * α^2
---
4
```

$\frac{9}{2}$

```
p2PotEnergy =
  Plot[PotEnergy, {x, -5, 5}, BaseStyle → {FontFamily → Helvetica, FontSize → 12, FontColor → RGBColor[0, 0.5, 0]},
    PlotRange → {-5, 5}, PlotPoints → 20, Frame → True, FrameStyle → White,
    FrameTicks → {{{{4.5, "E=1/4 kα^2"}}, {{4.5, "E=1/4 kα^2"}}}, {{{-3, "-α"}, {3, "α"}}, {{-3, "-α"}, {3, "α"}}}},
    FrameTicksStyle → Directive[Blue], GridLines → {{{-3, Blue}, {3, Blue}}, {{4.5, Blue}}},
    PlotStyle → {RGBColor[0, 0.5, 0]}, PlotLabel → "Potential Energy"]
```